LAST NAME:

FIRST NAME: Solution

Problem 1 Let $\Sigma = \{a, b, c, d\}$.

Let e be the regular expression defined as follows:

$$e = ab \cup cd \cup ac \cup bd \cup ad$$

Let L be the language defined by e.

In each of the cases below, state the cardinality of the given set. If this cardinality is finite, state the *exact number.* (An arithmetic expression is *not acceptable* as answer.) If this cardinality is infinite, state it and specify whether it is countable or uncountable.

(a) class of all languages over Σ

Answer: infinite, uncountable

(b) class of languages over Σ that are regular

Answer: infinite, countable

(c) class of languages over Σ that are not regular

Answer:

(d) L

Answer: 5

(e) $\mathcal{P}(\Sigma)$ (set of subsets of Σ)

Answer: 16

(f) $\mathcal{P}(L)$ (set of subsets of L)

Answer: 32

(g) class of languages over Σ that are finite

Answer: infinite, countable

(h) class of languages over Σ that are infinite

Answer: infinite, uncountable

(i) class of languages over Σ that have no finite description

Answer: infinite, uncountable

(j) set whose regular expression over Σ is:

$$\emptyset \cup a$$

Answer: 1

(k) set whose regular expression over Σ is:

$$\emptyset^* \cup a$$

Answer: 2

(l) set whose regular expression over Σ is:

$$\emptyset \cup e$$

Answer: 5

(m) set whose regular expression over Σ is:

$$\emptyset e$$

Answer: 0

(n) set whose regular expression over Σ is:

$$\emptyset \cup ee$$

Answer: 25

(o) set whose regular expression over Σ is:

$$\emptyset^* e$$

Answer: 5

(p) set whose regular expression over Σ is:

$$\lambda \cup e$$

Answer: 6

(q) set whose regular expression over Σ is:

$$\lambda e$$

Answer: 5

(r) set whose regular expression over Σ is:

$$\emptyset^* \cup \lambda^*$$

Answer: 1

(s) set whose regular expression over Σ is:

$$a\,e$$

Answer: 5

(t) set whose regular expression over Σ is:

$$e^*$$

Answer: infinite, countable

(u) set whose regular expression over Σ is:

$$(ee)^*$$

Answer: infinite, countable

LAST NAME:

FIRST NAME: Solution

Problem 2 Let L be the language defined by the regular expression:

$$((caaa)^* \cup (ad \cup c^*bab)^* \, (bc^*a)) \;\; (dda)^*$$

Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d\}$

$V = \{S, A, B, D, E, F, J\}$

P:

$S \to AB$

$A \to D \mid EF$

$D \to \lambda \mid DD \mid caaa$

$E \to \lambda \mid EE \mid ad \mid Jbab$

$J \to cJ \mid \lambda$

$F \to bJa$

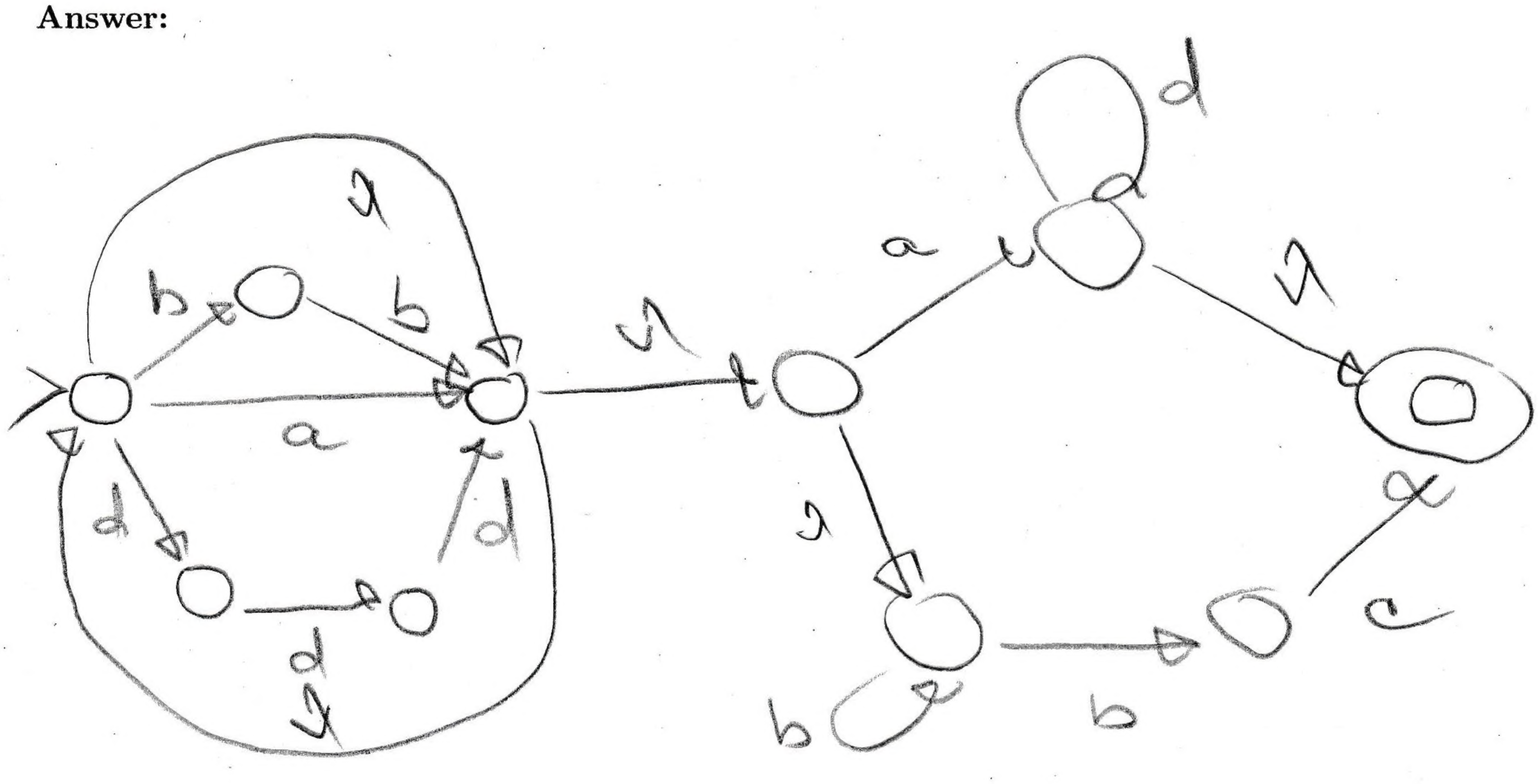
$B \to \lambda \mid BB \mid dda$

Problem 3 Let L be the language defined by the regular expression:

$$((a \cup bb \cup ddd)^*) \;\; ((ad^*) \cup (b^*bc))$$

Draw a state-transition graph of a finite-state automaton that accepts the language L. If such an automaton does not exist, prove it.

Answer:



LAST NAME: ______

FIRST NAME: Solution

Problem 4 Let L be the set of exactly those strings over the alphabet $\Sigma = \{a, b, c\}$ that contain as substring at least one of the following three strings: cca, bac, ab.

Write a regular expression that defines L. If such a regular expression does not exist, prove it.

Answer:

$$(a \cup b \cup c)^*(cca \cup bac \cup ab)(a \cup b \cup c)^*$$

Problem 5 Let L be the set of exactly those strings over the alphabet $\Sigma = \{a, b, c\}$ whose length is odd, and the middle symbol is equal to the first symbol and to the last symbol.

Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c\}$

$V = \{S, A, B, K, Z\}$

P:

$S \to aAa \mid bBb \mid cKc \mid a \mid b \mid c$

$A \to ZAZ \mid a$

$B \to ZBZ \mid b$

$K \to ZKZ \mid c$

$Z \to a \mid b \mid c$

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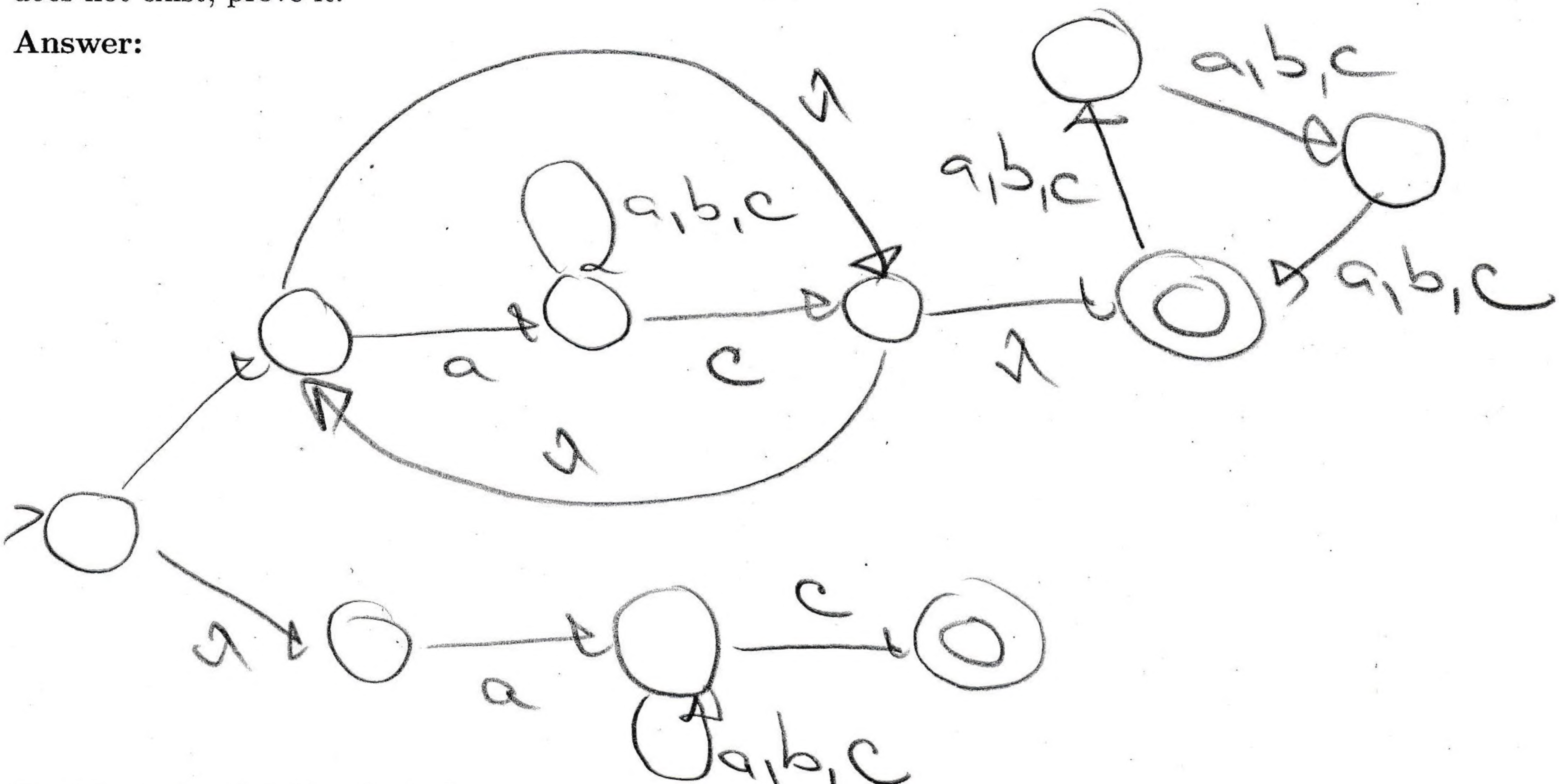
Problem 6 Let $\Sigma = \{a, b, c\}$.

Let L_1 be the set of exactly those strings over Σ that begin with a and end with c.

Let L_2 be the set of exactly those strings over Σ whose length is divisible by 3.

Draw a state-transition graph of a finite automaton that accepts the language $L_1^* L_2 \cup L_1$. If such an automaton does not exist, prove it.

Answer:



Problem 7 Let $\Sigma = \{a, b, c\}$.

Let L_1 be the set of exactly those strings over Σ where the number of c's is equal to 3.

Let L_2 be the set of exactly those strings over Σ that contain (in any order) the two substrings: aac, bbc.

Write a complete formal definition of a context-free grammar that generates the language $L_1^* \cup L_2^*$. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c\}$

$V = \{S, A, B, D, E\}$

P:

$S \to A \mid B$

$A \to \lambda \mid AA \mid DcDcDcD$

$D \to \lambda \mid DD \mid a \mid b$

$B \to \lambda \mid BB \mid EaacEbbcE \mid EbbcEaacE$

$E \to \lambda \mid EE \mid a \mid b \mid c$

LAST NAME:

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Problem 8 Let L be the language over the alphabet $\Sigma = \{a, b, c, d\}$ that contains exactly those strings whose form is:

$$b^i c^{j+3} a^\ell d^{m+2} b^{n+1} a^{p+2} c^q$$

where $i, j, \ell, m, n, p, q \geq 0$ are natural numbers such that: $i = m,\ n = q,\ j = \ell$

Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c, d\}$

$V = \{S, A, B, D, E\}$

P:

$S \to AB$

$A \to bAd \mid Ddd$

$D \to cDa \mid ccc$

$B \to bBc \mid bE$

$E \to aE \mid aa$

Problem 9 Let L be the language over the alphabet $\Sigma = \{a, b, c\}$ that contains exactly those strings which satisfy all of the following properties:

1. if the string does not contain any a's, then the string is a concatenation of at least three palindromes whose length is odd and greater than 1;
2. if the string contains at least one a, then the string is palindrome whose length is odd and the middle symbol is a.

Write a complete formal definition of a context-free grammar that generates the language L. If such a grammar does not exist, prove it.

Answer:

$G = (V, \Sigma, P, S)$

$\Sigma = \{a, b, c\}$

$V = \{S, S_1, S_2, A, B, Z\}$

P:

$S \to S_1 \mid S_2$

$S_1 \to AS_1 \mid AAA$

$A \to bAb \mid cAc \mid B$

$B \to bZb \mid cZc$

$Z \to b \mid c$

$S_2 \to aS_2a \mid bS_2b \mid cS_2c \mid a$

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Problem 10 Let L be the language over the alphabet $\Sigma = \{a, b, c\}$ that contains exactly those strings which satisfy all of the following properties:

1. the three symbols a, b, c occur in the string in the alphabetic order;
2. the substring which consists of a's is a non-empty palindrome whose length is even;
3. the substring which consists of b's is a palindrome whose length is odd;
4. the substring which consists of c's is not empty.

If L is regular, then use part (a) of the answer space below to write a regular expression that defines L, and do not write anything in part (b).

If L is not regular, then do not write anything in part (a) of the answer space, but complete the missing parts of the text given in part (b) so as to obtain a proof that L is not regular.

(a) Regular expression for L:

Answer: $(aa)^*aa(bb)^*bc^*c$

(b) Proof that L is not regular:

Observe that all words of L satisfy the following characteristic property:

Assume the opposite, that L is regular. Let π be the constant as in the Pumping Lemma for L. Let $w_0 \in L$ be a string defined as follows:

$w_0 =$

w_0 belongs to L because

w_0 must pump because

In any "pumping" decomposition of w_0, the pumping window satisfies the following property:

because

By pumping times, we obtain a string:

which violates the stated characteristic property because

and thus does not belong to L.

Since L violates the Pumping Lemma, L is not regular.